

# QCD Phase Transition in Brans-Dicke DGP model of Brane Gravity

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## Abstract

In this work, the quark-hadron phase transition in a chameleon Brans-Dicke model of brane world cosmology within an effective model of QCD is investigated. Whereas, in the chameleon Brans-Dicke model of brane world cosmology, the Friedmann equation and conservation of density energy are modified, resulting in an increased expansion in the early Universe. These have important effects on quark-hadron phase transitions. We investigate the evolution of the physical quantities relevant to quantitative descriptions of the early times, namely, the energy density,  $\rho$ , temperature,  $T$ , and the scale factor,  $a$ , before, during, and after the phase transition. We do this for smooth crossover formalism in which lattice QCD data is used for obtaining the matter equation of state and first order phase transition formalism. Our analyses show that the quark-hadron phase transition has occurred at approximately one nanosecond after the big bang and the general behavior of temperature is similar in both of two approaches.

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## 1 Introduction

General theory of relativity, the most beautiful physical theory, is a geometrical theory of spacetime. It successfully passed many experimental test related to solar system and its results are in great agreement with observational data. Moreover general theory of relativity, there are other theories describing gravitational phenomena of our Universe. Scalar tensor theory (STT) is one of these theories, first introduced by P. Jordan in 1955 [1]. Based on STT, in addition to metric there is a non-minimally coupled scalar field to gravity which contribute describing the gravity. It is able to produced a varying gravitational constant. Among STT, the most famous instance is Brans-Dicke theory proposed by C. Brans and R. H. Dicke in 1961 [2]. The starting point of Brans and Dicke was the idea of Mach, in which they put Mach idea in general relativity by inserting a non-minimally coupled scalar field into Einstein-Hilbert action describing varying gravitational constant. The theory might be able to generate an accelerating solution of the Universe evolution without including cosmological constant [3].

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Higher dimensional theory is another alternative theory of gravity. Among them, five-dimensional brane-world scenario received huge interest. This scenario present an fascinating picture of the Universe in which the standard matter and their interaction are confined to a four-dimensional hypersurface (the brane) embedded in five-dimensional spacetime (the bulk). The first model of five-dimensional brane-world was introduced by Randall and Sundrum in 1999 [4, 5]. They proposed two models inspired from string theory: their first model includes two brane with opposite brane tension and the second model contains a brane with positive tension which received cosmologist attention. The evolution equation of model is somewhat different with standard four-dimension. Quadratic energy density and bulk Weyl tensor contribute in evolution equation of brane and cause deviation from standard four-dimensional cosmology in high energy density regime. However standard cosmology equation is recovered in low energy limits [6, 7, 8]. In 2000, another model of brane-world introduced by Dvali, Gabadadze and Porrati (DGP) [9]. The main difference of this model with RS-brane-world is the presence of Einstein term in brane action as well as bulk action. Since the matter confined on brane is coupled with bulk gravity, some quantum correction are induced on brane action. The curvature scalar in brane action is the results of such a correction. It is well-known that the DGP model contain two branches related to two value of  $\epsilon$  parameter, which appears in Friedmann equation of the model. In brief and for original model of DGP, these two branches can be explained as bellow [10]:

- $\epsilon = -1$ : In late time, under some condition, the Friedmann equation could describe a fully five-dimension regime and have a transition from four-dimension regime to five-dimension regime. This branch of the DGP model is known as *normal branch* [4, 5].
- $\epsilon = +1$ : The late time behavior of this case is different. Friedmann equation never goes to five-dimension regime, and if we assume that the energy density decreases by passing time, the model describes an inflationary solution. Then,  $\epsilon = +1$  branch is able to give a quintessence-like scenario. This branch of the DGP model is known as *self-accelerating branch* [4, 5].

The DGP model has attracted much attention to study the different aspects of the evaluation of the Universe [9, 11, 10, 12, 13].

It is well known that DGP model is very successful in investigation of late time evaluation of the Universe. Therefore, we will study the QHP transition as an early Universe phase transition in the context of DGP brane-world senario with BD model of a perfect fluid on the brane. For more about early Universe phase transition, see Ref. [14, 15, 16, 17, 18, 19, 20]. As the Universe expanded and cooled it passed through a series of symmetry-breaking phase transitions which can generate topological defects. Although, for QHP transition a possibility of no phase transition was considered in Ref. [21, 22, 23, 24, 25], but it could be a first-, second-, or higher-order phase transition. The order of the phase transition depends strongly on the mass and flavor of the quarks. For an early study of a first-order QHP transition in the expanding Universe see Ref. [26]. As the deconfined quark-gluon plasma cools below the critical temperature  $T_c, \simeq 150$  MeV, it becomes energetically favorable to form color-confined hadrons (mainly pions and a few of neutrons and protons, due to the conserved net baryon number). However, this new phase does not form immediately. As it is characteristic of a first-order phase transition, some supercooling is needed to overcome the energy expense of forming the surface of the bubble and the new hadron phase. When a hadron bubble is nucleated, latent heat is released, and a spherical

shock wave expands into the surrounding supercooled quark-gluon plasma. This reheats the plasma to the critical temperature, preventing further nucleation in regions passed through by one or more shock fronts. Generally, bubble growth is described by deflagrations, with a shock front preceding the actual transition front. The nucleation stops when the Universe has reheated to  $T_c$ . After that, the hadron bubbles grow at the expense of the quark phase and eventually percolate or coalesce. The transition ends when all quark-gluon plasma has been converted to hadrons, neglecting possible quark nugget production [27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39].

The main purpose of this work is considering QHP transition in DGP model of gravity including BD scalar field on brane. As mentioned before, BD theory is an interesting alternative theory of gravity which overcomes some problems of general relativity in standard four-dimension cosmology. In this work, we are going to insert an STT in brane action instead of usual Einstein-Hilbert action. The evolution equation in this model comes into different form with standard cosmology. Therefore, one of basic equation in cosmological studies undergoes some changes and different results are expected for steps of the Universe evolution. By assuming such a framework, we study behavior of QHP transition and some related quantities in early times of the Universe evolution and compare the results with other works. The QHP transition will be studied for both branches of DGP model, and both of them have equal contribution in the present work. It seems that, the self-accelerating branch has ghost-instability problem, and studying QHP transition for this branch is not helpful. However, this instability problem occurs in quantum level, where the theory of gravity is unclear as well. Therefore, it might be so soon to leave the self-accelerating branch because of its ghost-instability problem. This is enough for cosmologist to still attend the self-accelerating branch, in which there could be found so many work about the various topic of this branch [40, 41, 42, 43, 44].

This paper is planned as following: In Sec. 2, we derive the basic equations of our model. In Sec. 3, we investigate the QHP transition by assuming the smooth crossover approach and consider the evolution of temperature for both high and low temperature regime. We study the QHP transition by considering first-order phase transition formalism and obtain the behavior of temperature before, during and after phase transition in Sec. 4. The results are summarized in Sec. 5, and compares with the previous results of other works.

## 2 General Framework

In brane-world scenario, we take a four-dimensional spacetime (the brane) as the Universe which has been embedded in five-dimensional spacetime (the bulk). We consider the DGP model of brane world scenario with the following action

$$\begin{aligned} \mathcal{S} &= S_{bulk} + S_{brane} \\ &= \int d^5x \sqrt{-\mathcal{G}} \left( \frac{\mathcal{R}}{2\kappa_5^2} - \Lambda \right) \\ &\quad + \int d^4x \sqrt{-g} \left( [\tilde{K}] + \frac{1}{2} \left[ \phi R - \frac{\omega}{\phi} \phi^\alpha \phi_\alpha \right] + \mathcal{L}_m \right), \end{aligned} \tag{1}$$

where  $\tilde{K} = [K]/\kappa_5^2$  and  $[K]$  is exterior curvature. The action is expressed in term of five-dimensional coordinates  $(x^0, x^1, x^2, x^3, y)$  on bulk, in which the brane is described by hypersurface  $y = 0$ . In the bulk action  $\mathcal{G}$  is the determinant of five-dimensional metric  $\mathcal{G}_{AB}$

with signature  $(- + + +)$ ,  $\mathcal{R}$  is Ricci scalar constructed from  $\mathcal{G}_{AB}$ ,  $\Lambda$  is five-dimensional cosmological.  $\kappa_5$  is a redefinition of five dimensional Newtonian gravitational constant  $G_5$  as  $\kappa_5^2 = 8\pi G_5$ , in which is related to five dimensional Plank mass by  $\kappa_5^2 = M_5^{-3}$ , and is taken as  $\kappa_5 = 1$  for convenience. In the brane action,  $g$  and  $R$  are four-dimensional determinant and Ricci scalar related to induced metric  $g_{\mu\nu} = \delta_\mu^A \delta_\nu^B \mathcal{G}_{AB}$  respectively.  $\phi$  is BD scalar field which lives on brane without potential term.  $\omega$  is a dimensionless coupling constant which determines the coupling between gravity and  $\phi$ . Lagrangian of brane-matter and brane-tension is denoted by  $\mathcal{L}_m$  which is defined as  $\mathcal{L}_m = L_m - \lambda$ .

The Einstein field equation is derived by taking variation of action with respect to metric as

$$^{(5)}G_{AB} = \mathcal{R}_{AB} - \mathcal{G}_{AB}\mathcal{R} = T_{AB}^{(\Lambda)} + T_{AB}\delta(y), \quad (2)$$

where  $T_{AB}^{(\Lambda)} = -\Lambda\mathcal{G}_{AB}$  describes the energy-momentum tensor of the bulk cosmological constant and  $T_{AB}$  is total brane energy-momentum tensor with following definition

$$T_{AB} = g_A^\mu g_B^\nu \left[ \phi T_{\mu\nu}^G + T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(m)} \right], \quad (3)$$

where  $T_{\mu\nu}^G$ , is due to presence of curvature scalar in brane action and behave as a source of gravity,  $T_{\mu\nu}^{(\phi)}$  is related to BD scalar field and  $T_{\mu\nu}^{(m)}$  is brane-matter energy-momentum tensor which are given by

$$\begin{aligned} T_{\mu\nu}^G &= -G_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}R - R_{\mu\nu}, \\ T_{\mu\nu}^{(\phi)} &= \frac{\omega}{\phi}(\phi_\mu\phi_\nu - \frac{1}{2}g_{\mu\nu}\phi_\alpha\phi^\alpha) + [\phi_{\mu;\nu} - g_{\mu\nu} - \Delta\phi], \\ T_{\mu\nu}^{(m)} &= \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \end{aligned} \quad (4)$$

where  $\Delta$  is D'Alembert with respect to brane metric.

On the other hand, taking variation of action with respect to scalar field, gives

$$\frac{2\omega}{\phi} - \Delta\phi - \frac{\omega}{\phi^2}\partial_\alpha\phi\partial^\alpha\phi + R = 0. \quad (5)$$

We consider a five-dimensional spatially flat FLRW metric describing by

$$ds_5^2 = -n^2(\tau, y)dt^2 + a^2(\tau, y)\delta_{ij}dx^i dx^j + dy^2. \quad (6)$$

Account this assumption, the components of Einstein field equation can be written as

$$3\left\{\frac{\dot{a}^2}{a^2} - n^2\left(\frac{a''}{a} + \frac{a'^2}{a^2}\right)\right\} = T_{00}^{(\Lambda)} + T_{00}\delta(y), \quad (7)$$

$$\begin{aligned} a^2\delta_{ij}\left[\frac{a'}{a}\left(\frac{a'}{a} + 2\frac{n'}{n}\right) + 2\frac{a''}{a} + \frac{n''}{n}\right] + \\ \frac{a^2\delta_{ij}}{n^2}\left[\frac{\dot{a}}{a}\left(-\frac{\dot{a}}{a} + 2\frac{\dot{n}}{n}\right) - 2\frac{\ddot{a}}{a}\right] = T_{ij}^{(\Lambda)} + T_{ij}, \end{aligned} \quad (8)$$

$$3\left(\frac{n'}{n}\frac{\dot{a}}{a} - \frac{\dot{a}'}{a}\right) = T_{05}^{(\Lambda)}, \quad (9)$$

$$3 \left[ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{1}{n^2} \left\{ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right\} \right] = T_{55}^{(\Lambda)} + T_{55}. \quad (10)$$

The metric components are continuous across the brane, however their first derivative with respect to  $y$  are discontinuous and second derivative contain Dirac delta function. Following [6, 7, 8] we write

$$\begin{aligned} n_0'' &= \hat{n}'' + [n]_0 \delta(y), \\ a_0'' &= \hat{a}'' + [a]_0 \delta(y), \end{aligned}$$

where  $\hat{n}''$  and  $\hat{a}''$  are the non-distributional part of  $n(\tau, y)$  and  $a(\tau, y)$  respectively. Substituting this definition in Eq.(7) and (8) one can derive the junction conditions as

$$\frac{[a']_0}{a_0} = -\frac{1}{3}T_{00}, \quad (11)$$

$$\frac{[n']_0}{n_0} = \frac{1}{3} \left[ 3a^{-2}T_{11} + 2T_{00} \right]. \quad (12)$$

We suppose the brane-matter energy-momentum tensor is filled with perfect fluid

$$T^{(m)\mu}_{\nu} = \delta(y) \text{diag}(-\rho_b, p_b, p_b, p_b, 0). \quad (13)$$

Inserting this expression in Eqs.(11) and (12), one can see that the junction conditions are compatible with [6, 7, 8]. The brane-matter energy density  $\rho_b$  and pressure  $p_b$  are defined by

$$\rho_b = \rho + \lambda; \quad p_b = p - \lambda, \quad (14)$$

where  $\rho$  and  $p$  are density and pressure of matter respectively.

## 2.1 Friedmann equation

The (0-0) and (5-5) component of Einstein equation in the bulk can be rewritten as the following form [6, 7, 8, 12],

$$\mathcal{F}' = \frac{2a^3 a'}{3} T^{(\Lambda)0}_0, \quad (15)$$

$$\dot{\mathcal{F}} = \frac{2a^3 \dot{a}}{3} T^{(\Lambda)5}_5, \quad (16)$$

where  $\mathcal{F}$  is defined by

$$\mathcal{F} = \left( \frac{aa'}{n} \right)^2 - (a\dot{a})^2. \quad (17)$$

The bulk cosmological constant is independent of time and fifth dimension. Then integrating of Eq.(15) leads one to the following result

$$\mathcal{F} = -\frac{a^4 \Lambda}{6} - \zeta \quad (18)$$

where  $\zeta$  is a constant of integration and can only be a function of time. Substituting Eq.(17) in Eq.(18), we arrive at

$$\left( \frac{\dot{a}}{a} \right)^2 = \left( \frac{a'}{an} \right)^2 + \frac{\Lambda}{6} + \frac{\zeta}{a^4} \quad (19)$$

Eq.(19) easily could be obtained on  $y = 0$ . For simplicity and without loss the generality, we assume  $n_0 = 1$ , where the index  $-0-$  indicates the quantities on hypersurface  $y = 0$ . Using  $Z_2$ -symmetry assumption ( $[a']_0/a_0 = 2a'_0/a_0$ ) and junction condition, lead one to the generalized Friedmann equation on the brane as

$$H^2 = \left(-\frac{T_{00}}{6}\right)^2 + \frac{\Lambda}{6} + \frac{\zeta}{a^4} \quad (20)$$

where  $H = \dot{a}_0/a_0$  is the Hubble parameter on brane. So that (0-0)-component of brane energy-momentum tensor is

$$T_{00} = \left[-3H^2\phi + \frac{\omega}{\phi}\frac{\dot{\phi}^2}{2} - 3H\dot{\phi} + (\rho + \lambda)\right] \quad (21)$$

Based on homogeneity and isotropy of the Universe we expect scalar field depends only on time,  $\phi = \phi(\tau)$ . To determine the exact function of scalar field, one should solve Eq.(6) where scalar field has a non-minimal coupling to geometry. Finding out an exact solution encounters difficulties. In these cases, according to [45, 46], we assume

$$\phi = \phi_0 a^\beta \quad (22)$$

this choice give the time derivative of scalar field as  $\dot{\phi} = \beta\phi H$ . Then Eq.(21) is rewritten as

$$T_{00} = \left(\frac{\beta^2\omega}{2} - 3(1 + \beta)\right)\phi H^2 + (\rho + \lambda), \quad (23)$$

and finally (0-0)-component of field equation is derived as

$$H^2 = \frac{\Lambda}{6} + \frac{1}{36} \left[3A\phi H^2 + (\rho + \lambda)\right]^2 + \frac{\zeta}{a^4} \quad (24)$$

where  $A = \beta^2\omega/6 - (1 + \beta)$ . The effective DGP length scale in this model could be defined as

$$l_{effDGP} = \frac{6}{|A|\phi}, \quad (25)$$

where  $\phi$  is a function of time, then the effective DGP length scale is varying by time. In this case, variation of scale factor was defined as Eq.(22). Therefore, one could introduced a constant DGP length scale as  $l_c = 6/|A|\phi_0$ .

The familiar form of Friedmann equation can be described as

$$H^2 = \frac{2}{\Phi^2} \left( \chi + \epsilon \sqrt{\chi^2 - \frac{\Phi^2}{36}(\rho_b^2 + 6\Lambda)} \right), \quad (26)$$

where  $\chi = (1 - (\Phi\rho_b)/6)$ ,  $\rho_b = \rho + \lambda$ ,  $\Phi = A\phi$  and  $\epsilon = \pm 1$ . Also the brane effective cosmological constant is described by  $\Lambda_4 = [\Lambda + (\kappa_5^2\lambda^2/6)]\kappa_5^2/6$ , which same as RS-model has been assumed to be zero. The last term which describes the dark radiaon is ignored as well.

The (0-5) component of field equation (9) shows that  $n'/n = \dot{a}'/\dot{a}$ . Inserting the junction conditions (11) and (12) leads one to the usual continuity equation for brane matter as

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (27)$$

Since brane matter has no direct coupling to the other component in the action, the continuity of brane matter was expected.

### 3 Lattice QCD phase transition

The phase transition could be first or second order phase transition, or it can only be a crossover with rapid change in the variable. The type of transition strongly depends on the values of quark masses. In this section we restrict the analysis to lattice QCD phase transition and consider the physical quantities related to the QHP transition in the context of DGP brane-world scenario with brane BD scalar field. Let's have a brief review on lattice QCD. Lattice QCD phase transition is one of fundamental concept in particle physics and is deeply relevant to any study corresponding to the early times evolution of the universe. Based on this scenario, there is a soup of quark and gluons which interact and hadron forms by crossover transition. Lattice QCD is a new approach which enable us to study nonperturbative regime of the QCD equation of state. Various methods exist for obtaining equation of state. The equation of state has been estimated in recent calculation for 2+1 flavor QCD. The most extensive estimation of equation of state has been performed with fermion formulation on lattice with temporal extent  $N_t = 4, 6$  [47, 48, 49],  $N_t = 8$  [50] and  $N_t = 6, 8, 10$  [51]. The trace anomaly can be calculated accurately for high temperature region  $T > 250\text{MeV}$ . Therefore, in order to construct a realistic equation of state in high temperature region, one can use lattice data for the trace anomaly in this region.

The condition is somehow different for low energy regime. In low energy regime where  $T < 180\text{MeV}$ , the trace anomaly is affected by large discretization effect. But, the hadronic resonance gas (HRG) model is suitable to construct equation of state in low energy regime and has been well studied [52].

#### 3.1 High temperature regime

As mentioned before, in high temperature regime where  $T > 250\text{MeV}$ , the trace anomaly can be applied precisely to derive an equation of state. In this regime, the gluon and quarks effectively are massless and behave like a radiation. The data are fit on a simple form of equation of state as

$$\rho(T) \simeq \alpha T^4, \quad (28)$$

$$p(T) \simeq \sigma T^4, \quad (29)$$

where  $\alpha = 14.9702 \pm 009997$  and  $\sigma = 4.99115 \pm 004474$  are found out using a least-squares fit [48, 49]. Substituting these relations in conservation relation (27) we have

$$a(T) = cT^{-\frac{4\alpha}{3(\alpha+\sigma)}}, \quad (30)$$

where  $c$  is a constant of integration. Substituting Eq.(30) into Eq.(26), the time evolution of temperature in this regime is derived as

$$\begin{aligned} \dot{T} &= -\frac{3(\alpha + \sigma)T}{4\alpha} \\ &\times \left[ \frac{2}{\Phi^2} \left( \chi + \epsilon \sqrt{\chi^2 - \frac{\Phi^2}{36}(\rho_b^2 - \lambda^2)} \right) \right]^{1/2}. \end{aligned} \quad (31)$$

This expression describes the behavior of temperature as a function of time. We have plotted the numerical results of Eq.(31) in figure 1 for normal branch ( $\epsilon = -1$ ) and in figure 2 for self-accelerating branch ( $\epsilon = +1$ ). The figures display the effective temperature in

the QGP in BD model of DGP brane-world for temperature  $T > 250\text{MeV}$ , acquired for the smooth crossover approach. It seems that, we almost have the same consequences for both branches of the model. It is realized that, the Universe becomes cooler by passing time and the QGP is finished at about 5 – 7 micro-second after the Big Bang.

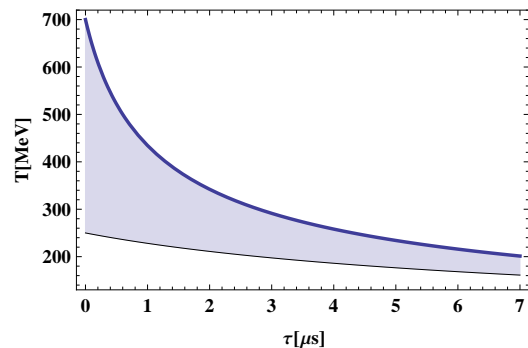


Figure 1:  $T$  versus  $\tau$  according to high temperature region of the smooth crossover procedure in the DGP brane gravity with BD scalar field in brane. The constant parameters are taken as:  $\beta = 2.47 \times 10^{-3}$ ,  $\phi_0 = 2 \times 10^5 \text{MeV}^2$ ,  $\omega = 10^4$  and  $\lambda = 10^9 \text{MeV}^4$ .

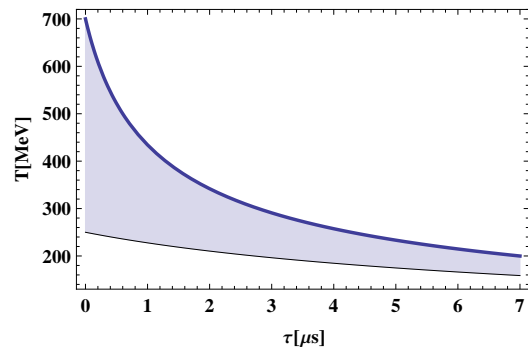


Figure 2:  $T$  versus  $\tau$  according to high temperature region of the smooth crossover procedure in the DGP brane gravity with BD scalar field in brane. The constant parameters are taken as:  $\beta = 2.47 \times 10^{-3}$ ,  $\phi_0 = 2 \times 10^5 \text{MeV}^2$ ,  $\omega = 10^4$  and  $\lambda = 10^9 \text{MeV}^4$ .

### 3.2 Low temperature regime

In contrast to the high temperature, the trace anomaly is not a suitable approach to estimate equation of state in low temperature regime. However, using the hadronic resonances gas (HRG) model, one can achieve to an equation of state in low temperature regime, where  $T < 180\text{MeV}$  [52]. In HRG model, the confinement of QCD is treated as non-interacting gas of fermions and bosons [53, 54]. The idea of the HRG model is to implicitly account for the strong interaction in the confinement phase by looking at the hadronic resonances only, since these are basically the only relevant degrees of freedom in that phase. It has been shown that, the HRG results can be parametrized for trace anomaly as [52]:

$$\frac{I(T)}{T^4} \equiv \frac{\rho - 3p}{T^4} = a_1 T + a_2 T^3 + a_3 T^4 + a_4 T^{10}, \quad (32)$$



where  $a_1 = 4.654 \text{ GeV}^{-1}$ ,  $a_2 = -879 \text{ GeV}^{-3}$ ,  $a_3 = 8081 \text{ GeV}^{-4}$ ,  $a_4 = -7039000 \text{ GeV}^{-10}$ , and  $I(T) = \rho(T) - 3p(T)$  is the trace anomaly. Through the calculation of trace anomaly  $I(T)$  in lattice QCD, the energy density, pressure and entropy could be estimated by using usual thermodynamics identities. Integral of trace anomaly displays the pressure difference between two temperature  $T$  and  $T_{\text{low}}$

$$\frac{p(T)}{T^4} - \frac{p(T_{\text{low}})}{T_{\text{low}}^4} = \int_{T_{\text{low}}}^T \frac{dT'}{T'^5} I(T'). \quad (33)$$

Due to exponential suppression,  $p(T_{\text{low}})$  could be ignored for sufficiently small values of the lower integration limit [55]. The energy density which is expressed as  $\rho(T) = I(T) + 3p(T)$  could be calculated. At the end, by using Eqs.(32) and (33) we have following relations for energy density and pressure respectively

$$\rho(T) = 3\eta T^4 + 4a_1 T^5 + 2a_2 T^7 + \frac{7a_3}{4} T^8 + \frac{13a_4}{10} T^{14}, \quad (34)$$

$$p(T) = \eta T^4 + a_1 T^5 + \frac{a_2}{3} T^7 + \frac{a_3}{4} T^8 + \frac{a_4}{10} T^{14}, \quad (35)$$

where  $\eta = -0.112$ . In this case, the era before the phase transition (namely quark-gluon phase) is considered at low temperature. Then the Universe is in confinement phase and could be treated as a non-interacting gas of fermions and bosons. Substituting Eqs.(34) and (35) in the conservation equation, the Hubble parameter can be derived as

$$H = -\frac{12\eta T^3 + 20a_1 T^4 + A(T)}{3[4\eta T^4 + 5a_1 T^3 + B(T)]} \dot{T}, \quad (36)$$

where we have

$$A(T) = 14a_2 T^6 + 14a_3 T^7 + \frac{91}{5} a_4 T^{13},$$

$$B(T) = \frac{7}{3} a_2 T^7 + 2a_3 T^8 + \frac{7}{5} a_4 T^{14}.$$

To obtain the scale factor as a function of temperature, we should integrate Eq.(36), then we have

$$a(T) = \frac{c}{T[60\eta + 75a_1 T + 35a_2 T^3 + 30a_3 T^4 + 21T^{10}]^{1/3}}, \quad (37)$$

here  $c$  is constant of integration. Finally using Eq.(36) and the Friedmann equation (26), one can find out the time evolution of temperature

$$\dot{T} = -\frac{3[4\eta T^4 + 5a_1 T^3 + B(T)]}{12\eta T^3 + 20a_1 T^4 + A(T)} \times \left[ \frac{2}{\Phi^2} \left( \chi + \epsilon \sqrt{\chi^2 - \frac{\Phi^2}{36}(\rho_b^2 - \lambda^2)} \right) \right]^{1/2}. \quad (38)$$

Differential Eq.(38) describes the behavior of temperature as a function of cosmic time in the QGP for DGP brane-world scenario with a BD scalar field in brane. The relation displays the era before phase transition at low temperature where the Universe is treated as a non-interacting gas of fermions and bosons since it is in confinement phase. Eq.(38) is solved numerically and its corresponding results are plotted for both  $\epsilon = -1$  and  $\epsilon = +1$

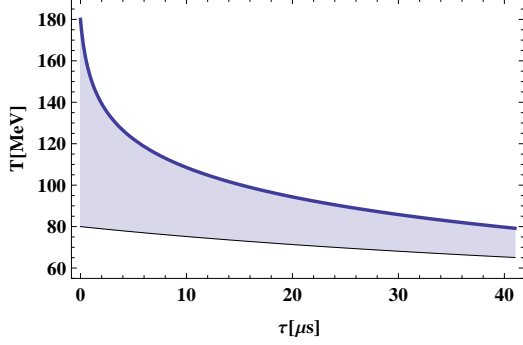


Figure 3:  $T$  versus  $\tau$  according to the low temperature region of the smooth crossover procedure in the DGP brane gravity with BD scalar field in brane for following constant parameters:  $\beta = 2.47 \times 10^{-3}$ ,  $\phi_0 = 2 \times 10^5 \text{MeV}^2$ ,  $\omega = 10^4$  and  $\lambda = 10^9 \text{MeV}^4$ .

respectively in figure 3 and figure 4 in interval temperature as  $50 \text{MeV} \leq T \leq 180 \text{MeV}$ . The figures display the behavior of temperature versus cosmic time. It can be realized that for low temperature regime, the QGP of the Universe is finished at about  $35 - 40$  micro-second after the Big Bang for  $\epsilon = -1$  of the model, and at about  $25 - 30$  micro-second after big bang for  $\epsilon = 1$ . For low temperature in smooth crossover approach, QGP takes places after high temperature regime which is in agreement with lattice QCD data analysis prediction. Also it is clearly seen that, phase transition for  $\epsilon = -1$  case occurs later than  $\epsilon = 1$  case.

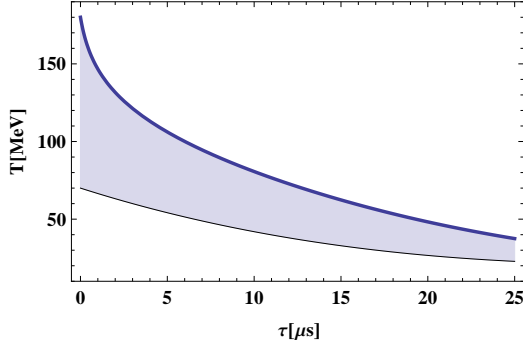


Figure 4:  $T$  versus  $\tau$  according to the low temperature region of the smooth crossover procedure in the DGP brane gravity with BD scalar field in brane for following constant parameters:  $\beta = 2.47 \times 10^{-3}$ ,  $\phi_0 = 2 \times 10^5 \text{MeV}^2$ ,  $\omega = 10^4$  and  $\lambda = 10^9 \text{MeV}^4$ .

## 4 First order quark hadron phase transition

The HRG model is no longer reliable in intermediate temperature  $180 \text{ MeV} < T < 250 \text{ MeV}$  because of large discretization effect in lattice calculation. So, the evolution of the Universe should be investigated again in this temperature interval. As mentioned before, it has been shown that the quark-hadron phase transition in QCD is characterized by the singular behavior of the partition function, and may be the first or second order phase transition [56]. In this section, we assume that the quark-hadron phase transition is first-order and occurs in the intermediate temperature regime  $T < 250 \text{MeV}$ . For a review of a

first-order quark-hadron phase transition, see Ref. [57] and references therein. Based on [56], the equation of state for matter in quark-gluon phase is given by

$$\rho_q = 3a_q T^4 + W(T), \quad p_q = a_q T^4 - W(T), \quad (39)$$

here, the subscript  $q$  denotes quark-gluon matter and the constant  $a_q = 61.75(\pi^2/90)$ . The potential energy density  $W(T)$  is [56]:

$$W(T) = B + \gamma_T T^2 - \alpha_T T^4, \quad (40)$$

where  $B$  is the bag pressure constant,  $\alpha_T = 7\pi^2/20$ , and  $\gamma_T = m_s^2/4$ , where  $m_s$ , the mass of the strange quark, is in the (60 – 200) MeV range. This form of  $W$  comes from a model in which the quark fields interact with a chiral field formed by the  $\pi$  meson field together with a scalar field. Results obtained from low energy hadron spectroscopy, heavy ion collisions, and from phenomenological fits of light hadron properties give a value of  $B^{1/4}$  between 100 and 200 MeV [57].

The cosmological fluid is usually taken as ideal gas of massless pions and nucleons in hadron phase, in which they can be described by Maxwell-Boltzmann distribution function with energy density  $\rho_h$  and pressure  $p_h$ . Then, the equation of state can be read as

$$p_h = \frac{1}{3}\rho_h = a_\pi T^4. \quad (41)$$

where  $a_\pi = 17.25\pi^2/90$ . On the other hand we have critical temperature  $T_c$  which is defined by the condition  $p_q(T_c) = p_h(T_c)$  [21, 22, 23, 24, 25]. Taking  $m_s = B^{1/4} = 200$ , the critical temperature in the presence model is given by

$$T_c = \left[ \frac{\gamma_T + \sqrt{\gamma_T^2 + 4B(a_q + \alpha_T - a_\pi)}}{2(a_q + \alpha_T - a_\pi)} \right]^{1/2} \approx 125 \text{ MeV}. \quad (42)$$

All physical quantities, such as the energy density, pressure, and entropy, exhibit discontinuities across the critical curve which is due to this fact that the phase transition is first-order. The main quantities which dealing through phase transition are energy density, pressure, scale factor and temperature. These quantities are determined using conservation equation, Friedmann equation and equation of state. Now, we study the evolution of this quantities before, during and after phase transition in the context of DGP brane-world scenario with a BD scalar field in brane.

#### 4.1 Behavior of temperature in the quark-gluon phase (QGP) for general $W(T)$

Before the transition, where  $T > T_c$ , the fluid is in quark phase describing by Eq.(41) as the equation of state. Inserting this equation of state in the conservation equation (27) and using the potential energy density of Eq.(40), the Hubble parameter is read as

$$H = - \left[ \frac{3a_q - \alpha_T}{3a_q} + \frac{\gamma_T}{6a_q T^2} \right] \frac{\dot{T}}{T}. \quad (43)$$

Integrating above relation gives the scale factor as

$$a(T) = cT^{\frac{\alpha_T - 3a_q}{3a_q}} \exp \left( \frac{\gamma_T}{12a_q T^2} \right). \quad (44)$$

Then, substituting Eq.(43) in the Friedmann Eq.(26), one can easily obtain the time evolution of temperature as

$$\begin{aligned} \dot{T} = & -\frac{6a_q T^3}{2(3a_q - \alpha_T)T^2 + \gamma_T} \\ & \times \left[ \frac{2}{\Phi^2} \left( \chi + \epsilon \sqrt{\chi^2 - \frac{\Phi^2}{36}(\rho_b^2 - \lambda^2)} \right) \right]^{1/2}. \end{aligned} \quad (45)$$

This relation displays the behavior of temperature in term of cosmic time. The numerical results of Eq.(45) are depicted in figure 5 for normal branch ( $\epsilon = -1$ ) and in figure 6 for self-accelerating branch ( $\epsilon = +1$ ). Figures show that the effective temperature in quark phase decrease by passing time. Enhancement of BD coupling constant increases the reduction rate of temperature. The temperature reaches to the critical temperature, where  $T = T_c \approx 125\text{MeV}$ , at about 10–25 micro-second after the Big-Bang for  $\epsilon = -1$ , and at about 6–22 micro-second after the Big Bang for  $\epsilon = 1$ . The temperature for  $\epsilon = 1$  reaches to its critical value earlier than  $\epsilon = -1$  case.

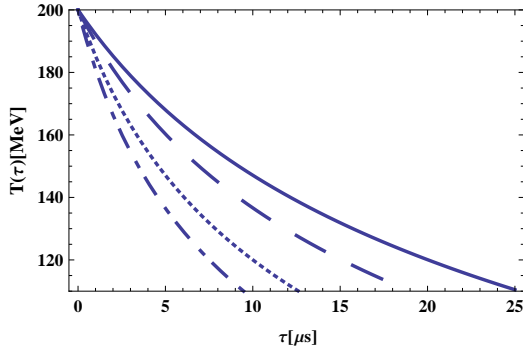


Figure 5:  $T$  versus  $\tau$  in the quark-gluon phase has been plotted for different values of BD coupling constant  $\omega$  as:  $\omega = 2.1 \times 10^3$  (solid line),  $\omega = 2.3 \times 10^3$  (dashed line),  $\omega = 2.5 \times 10^3$  (dotted line),  $\omega = 2.7 \times 10^3$  (dotted-dashed line). The other constant parameters are taken as:  $\beta = 5.24 \times 10^{-2}$ ,  $\phi_0 = 2 \times 10^4 \text{MeV}^2$ ,  $\lambda = 10^9 \text{MeV}^4$  and  $B^{1/4} = 200 \text{MeV}$ .

#### 4.1.1 Evolution of temperature in the QGP for $W(T) = B$

Another interesting model that deals with quark confinement, is that of an elastic bag which allows the quarks to move around freely. In this case the effect of temperature is ignored, so the potential  $W(T)$  is constant. As a result of this potential, equation of state obtain a simple form  $p_q = (\rho_q - 4B)/3$ , given by the bag model. Using conservation equation, one can derive

$$H = -\frac{\dot{T}}{T}; \quad a(T) = \frac{c}{T}. \quad (46)$$

Inserting Eq.(46) in the Friedmann equation, the time evolution of temperature in this case is derived easier than previous case which can be read as

$$\dot{T} = -T \left[ \frac{2}{\Phi^2} \left( \chi + \epsilon \sqrt{\chi^2 - \frac{\Phi^2}{36}(\rho_b^2 - \lambda^2)} \right) \right]^{1/2} \quad (47)$$

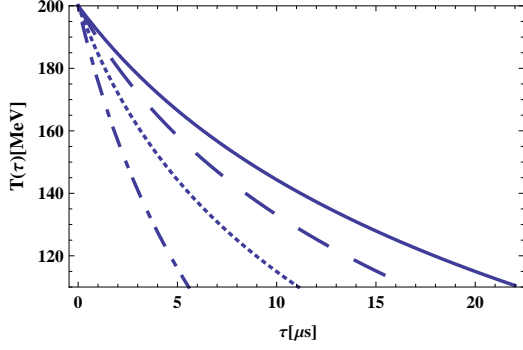


Figure 6:  $T$  versus  $\tau$  in the quark-gluon phase has been plotted for different values of BD coupling constant  $\omega$  as:  $\omega = 2.1 \times 10^3$  (solid line),  $\omega = 2.3 \times 10^3$  (dashed line),  $\omega = 2.5 \times 10^3$  (dotted line),  $\omega = 2.7 \times 10^3$  (dotted-dashed line). The other constant parameters are taken as:  $\beta = 5.24 \times 10^{-2}$ ,  $\phi_0 = 2 \times 10^4 \text{ MeV}^2$ ,  $\lambda = 10^9 \text{ MeV}^4$  and  $B^{1/4} = 200 \text{ MeV}$ .

Eq.(47) is solved numerically and the results are plotted in Figs.(7) and (8) for both branches. The figures display the behavior of temperature versus cosmic time for brane tension  $\lambda$ , and different values of BD coupling constant  $\omega$ . It can be found out that the decreasing of temperature in this case is nearly as the same as of the general case of  $W(T)$ . The temperature decreases by passing time and reaches to the critical values at about 9–23 micro-second after the big bang for the case  $\epsilon = -1$ , figure 7, and at about 5–21 micro-second after the Big Bang for the case  $\epsilon = +1$ , figure 7. It could be realized that for larger values of BD coupling constant, the time of transition in constant self-potential case occurs before the general case of self-potential.

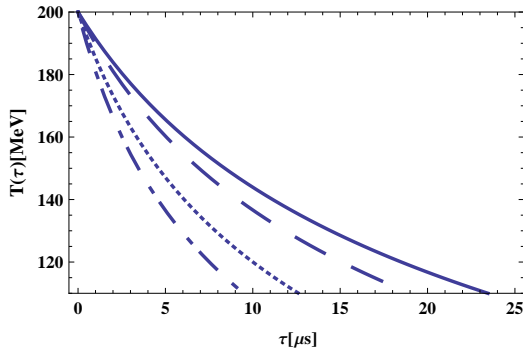


Figure 7: Temperature  $T$ , as a function of time,  $t$ , in the quark-gluon phase has been plotted for constant self-interacting potential  $W(T) = B$ , and for different values of BD coupling constant  $\omega$  as:  $\omega = 2.1 \times 10^3$  (solid line),  $\omega = 2.3 \times 10^3$  (dashed line),  $\omega = 2.5 \times 10^3$  (dotted line),  $\omega = 2.7 \times 10^3$  (dotted-dashed line). The other constant parameters are taken as:  $\beta = 5.24 \times 10^{-2}$ ,  $\phi_0 = 2 \times 10^4 \text{ MeV}^2$ ,  $\lambda = 10^9 \text{ MeV}^4$  and  $B^{1/4} = 200 \text{ MeV}$ .

## 4.2 Behavior of the hadron volume fraction

The temperature, pressure, enthalpy and entropy are conserved during phase transition, however quark matter density decreases from  $\rho_Q$  to  $\rho_H$ , the hadron matter density. During

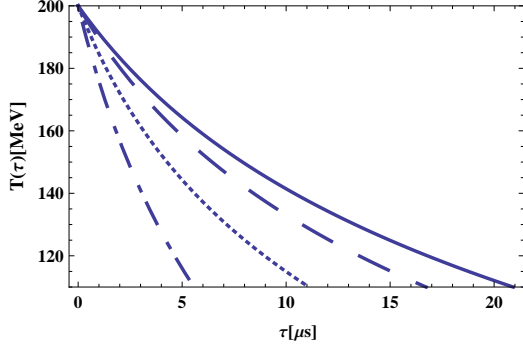


Figure 8: Temperature  $T$ , as a function of time,  $t$ , in the quark-gluon phase has been plotted for constant self-interacting potential  $W(T) = B$ , and for different values of BD coupling constant  $\omega$  as:  $\omega = 2.1 \times 10^3$  (solid line),  $\omega = 2.3 \times 10^3$  (dashed line),  $\omega = 2.5 \times 10^3$  (dotted line),  $\omega = 2.7 \times 10^3$  (dotted-dashed line). The other constant parameters are taken as:  $\beta = 5.24 \times 10^{-2}$ ,  $\phi_0 = 2 \times 10^4 \text{ MeV}^2$ ,  $\lambda = 10^9 \text{ MeV}^4$  and  $B^{1/4} = 200 \text{ MeV}$ .

this transition, we have  $T_c = 125 \text{ MeV}$ ,  $\rho_Q = 5 \times 10^9 \text{ MeV}^4$ ,  $\rho_H \approx 1.38 \times 10^9 \text{ MeV}^4$ , and constant pressure  $p_c \approx 4.6 \times 10^8 \text{ MeV}^4$ . Based on [57, 58, 60, 61], the energy density  $\rho(t)$  can be replaced by volume fraction of matter in hadron phase  $h(t)$  defining by following expression

$$\rho = \rho_H h(t) + [1 - h(t)] \rho_Q, \quad (48)$$

where  $\rho_H$  and  $\rho_Q$  respectively are energy density of hadron and quark. At the beginning of transition, all of matter are in quark phase, namely  $\rho(t_c) = \rho_Q$ ,  $h(t) = 0$ . By passing time, the phase of matter changes in which at the end of transition, the Universe enters in hadron phase and matter stay in hadron phase, namely  $\rho(t_h) = \rho_H$ ,  $h(t_h) = 1$ . Using Eq.(48) in the conservation relation results into

$$H = -\frac{1}{3} \frac{(\rho_H - \rho_Q) \dot{h}}{\rho_Q + p_c + (\rho_H - \rho_Q) h} = -\frac{1}{3} \frac{r \dot{h}}{1 + r h}, \quad (49)$$

where  $r = (\rho_H - \rho_Q)/(\rho_Q + p_c)$ . By integrating Eq.(49) one can easily obtain scale factor as

$$a(t) = a_0 [1 + r h]^{-1/3}. \quad (50)$$

The scale factor of the universe during the QHPT as a function of hadron volume fraction,  $h$ , which is illustrated in Fig.(9) shows that the universe expands in this era, although the temperature, pressure, enthalpy and entropy are constant. The hadron fraction increases by passing time and its time evolution is derived by inserting Eq.(49) in the Friedmann equation (26). Then one gets following expression as the time evolution of volume fraction

$$\begin{aligned} \dot{h} &= -\frac{3(1 + r h)}{r} \\ &\times \left[ \frac{2}{\Phi^2} \left( \chi + \epsilon \sqrt{\chi^2 - \frac{\Phi^2}{36} (\rho_b^2 - \lambda^2)} \right) \right]^{1/2}. \end{aligned} \quad (51)$$

The numerical results of differential Eq.(51) is presented in Figs.(10) and (11) for different values of BD coupling constant  $\omega$ . figure 10 is related  $\epsilon = -1$ , and figure 11 is represents

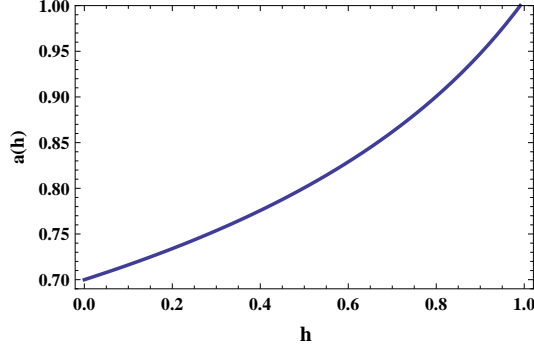


Figure 9: Scale factor as a function of the hadron volume fraction during the QHPT in the DGP brane gravity with BD scalar field in brane.

the results of  $\epsilon = +1$  case. The figure expresses the evolution of hadron volume fraction in terms of cosmic time during quark-hadron phase transition. Higher value of BD coupling constant increases the rate of phase transition which occurs at about 14 – 35 micro-second after the Big Bang for  $\epsilon = -1$ , and at about 7 – 25 micro-second after the Big Bang for  $\epsilon = 1$  case. The result is in complete agreement with the results of two previous cases. It is clear that, the hadron volume fraction parameter for  $\epsilon = -1$  case, reaches to one later than its corresponding for  $\epsilon = 1$  case. Then, the Universe in the case  $\epsilon = +1$  goes to pure hadronic phase earlier than the case  $\epsilon = -1$ .

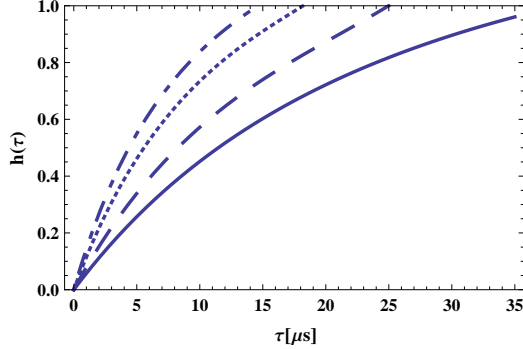


Figure 10: Hadron volume fraction as a function of cosmic time  $\tau$  has been depicted for different values of BD coupling constant  $\omega$  as:  $\omega = 2.1 \times 10^3$  (solid line),  $\omega = 2.3 \times 10^3$  (dashed line),  $\omega = 2.5 \times 10^3$  (dotted line),  $\omega = 2.7 \times 10^3$  (dotted-dashed line). The other constant parameters are taken as:  $\beta = 5.24 \times 10^{-2}$ ,  $\phi_0 = 2 \times 10^4 \text{ MeV}^2$ ,  $\lambda = 10^9 \text{ MeV}^4$  and  $B^{1/4} = 200 \text{ MeV}$ .

### 4.3 Evolution of temperature in Pure hadronic phase era

Finally, at the end of phase transition, the Universe is in the pure hadronic phase and the matter is described by a simple equation of state which is written as

$$\rho_h = 3p_h = 3a_\pi T^4. \quad (52)$$

Inserting Eq.(52) in conservation equation leads to the following result for Hubble parameter and scale factor

$$H = -\frac{\dot{T}}{T}; \quad a(T) = c \frac{T_c}{T}. \quad (53)$$

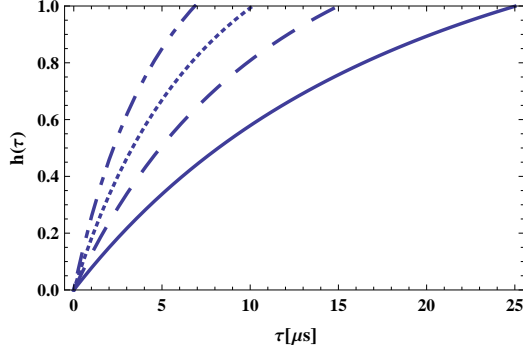


Figure 11: Hadron volume fraction as a function of cosmic time  $\tau$  has been depicted for different values of BD coupling constant  $\omega$  as:  $\omega = 2.1 \times 10^3$  (solid line),  $\omega = 2.3 \times 10^3$  (dashed line),  $\omega = 2.5 \times 10^3$  (dotted line),  $\omega = 2.7 \times 10^3$  (dotted-dashed line). The other constant parameters are taken as:  $\beta = 5.24 \times 10^{-2}$ ,  $\phi_0 = 2 \times 10^4 \text{MeV}^2$ ,  $\lambda = 10^9 \text{MeV}^4$  and  $B^{1/4} = 200 \text{MeV}$ .

Substituting above equation in the Friedmann Eq.(26) gives the time evolution of temperature in the pure hadronic phase of the Universe

$$\dot{T} = -T \left[ \frac{2}{\Phi^2} \left( \chi + \epsilon \sqrt{\chi^2 - \frac{\Phi^2}{36}(\rho_b^2 - \lambda^2)} \right) \right]^{1/2}. \quad (54)$$

The differential Eq.(54) is solved numerically and the results are plotted in Figs.(12) and (13) for different values of  $(\beta, \omega)$  in which  $\beta\omega = \text{cte}$  and is in order 100. The case  $\epsilon = -1$  is displayed in figure 12, and  $\epsilon = +1$  is presented in figure 13. It is found out that the effective temperature of the Universe decreases by passing time. The hadronic phase era of the Universe evolution for  $\epsilon = -1$  is occurred at about 42 – 84 micro-second after the Big Bang, and for  $\epsilon = 1$  case it is occurred at about 31 – 61 microsecond after the Big Bang. The acquired results are in agreement with the previous results related to the other era. It could be seen that for higher values of  $\beta\omega$  there is an enhancement in the rate of transition.

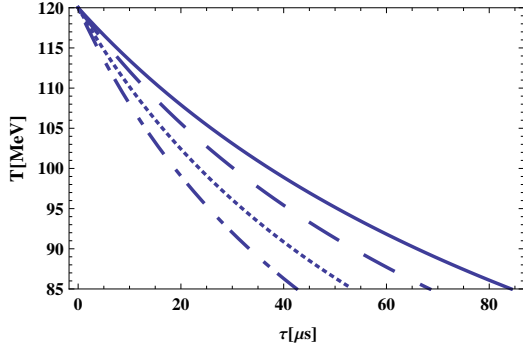


Figure 12: Evolution of temperature  $T$  versus  $t$  in the pure hadronic phase era has been depicted for different values of BD coupling constant  $\omega$  and  $\beta$  as:  $\beta\omega = 115.277$  (solid line),  $\beta\omega = 120.504$  (dashed line),  $\beta\omega = 125.506$  (dotted line),  $\beta\omega = 130.312$  (dotted-dashed line). The other constant parameters are taken as:  $\phi_0 = 2 \times 10^4 \text{MeV}^2$  and  $\lambda = 7 \times 10^{10} \text{MeV}^4$ .



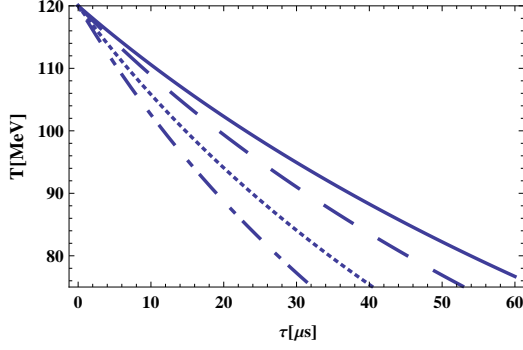


Figure 13: Evolution of temperature  $T$  versus  $t$  in the pure hadronic phase era has been depicted for different values of BD coupling constant  $\omega$  and  $\beta$  as:  $\beta\omega = 115.277$  (solid line),  $\beta\omega = 120.504$  (dashed line),  $\beta\omega = 125.506$  (dotted line),  $\beta\omega = 130.312$  (dotted-dashed line). The other constant parameters are taken as:  $\phi_0 = 2 \times 10^4 \text{MeV}^2$  and  $\lambda = 7 \times 10^{10} \text{MeV}^4$ .

## 5 Discussion and conclusion

In this work, we have investigated quark-hadron phase transition in the Brans-Dicke DGP model of Brane Gravity by considering two different formalism. We studied evolution of physical quantities relevant to physical description of the early times such as energy density, scale factor, and temperature, for two cases of  $\epsilon = -1$  and  $\epsilon = +1$  which are known as normal and self-accelerating branch respectively in original DGP model. At first, we studied QCD phase transition using smooth crossover approach for two regimes as high and low temperature. In High temperature regime, where  $T > 250 \text{MeV}$ , the trace anomaly is computed accurately and expressed a radiation like behavior. However, in low temperature regime, where  $T < 150 \text{MeV}$ , trace anomaly is effected by large discretization effect. Instead, HRG model is utilized to construct a realistic equation of state. Time of phase transition in high temperature regime is same for both cases of  $\epsilon = -1$  and  $\epsilon = +1$ , and occurs at about  $5 - 7$  micro-second after the Big Bang. In low temperature regime, phase transition occurs at about  $35 - 40$  micro-second after the Big Bang for case of  $\epsilon = -1$ , and at about  $20 - 25$  micro-second after the Big Bang for case of  $\epsilon = +1$ . As it was expected, time of transition in low temperature regime for both cases is occurs after the time of phase transition in high temperature regime. This result show the compatibility of these two consequences.

The results of this case could be compared with [58, 59, 60, 61]. In [58], the authors investigated QHP transition in RS brane-world model including a BD scalar field and in [59] for a self-accelerating branch of DGP brane scenario. They found out a transition time at about few micro-second after the big bang for both regimes.

Last, phase transition is investigated using first-order approach including three stages as: before phase transition (quark-gluon phase), during and after phase transition (Hadron phase). In first stage, we have two different models; general potential energy density  $W(T)$  and bag model with constant potential energy density,  $W(T) = \text{cte}$ . For both cases, Quark-gluon phase finished at about  $10 - 25$  ( $6 - 22$ ) micro-second after the Big Bang for case of  $\epsilon = -1$  ( $\epsilon = +1$ ); however quark-gluon phase in bag model finished earlier than general case of self-potential, which is more clear for higher values of BD coupling constant. At next stage, matter is a mixing of quark and hadron. The results showed that matter from a pure quark phase comes to a pure hadron phase at about  $14 - 35$  ( $7 - 25$ ) micro-second after the Big Bang for case of  $\epsilon = -1$  ( $\epsilon = +1$ ), in which later times is related to higher values of BD

coupling constant. After that, the matter comes to a pure hadronic phase era and behave like radiation. The transition completes at about  $42 - 84$  ( $31 - 61$ ) micro-second after the Big Bang for case of  $\epsilon = -1$  ( $\epsilon = +1$ ), which is in agreement with the results of previous cases. The transition occurs later for higher values of BD coupling constant which could be resulted that the larger value of BD coupling constant postpone the time of transition.

In this case, our results could be compared with [57, 58, 59, 60, 61]. The authors utilized a RS brane-world framework and considered their results for different values of brane tension [57]. It is shown that phase transition occurs at about micro-second after the Big Bang however earlier than our work. Also, they considered brane tension effect on the time of transition which results into later transition for larger value of brane tension. In [58] the authors have considered the phase transition in Brans-Dicke brane gravity and their results are plotted for different values of Brans-Dicke coupling constant, which displays that transition time increases by decreasing of BD coupling constant. The authors have considered the phase transition using a DGP brane scenario and their results are depicted for different values of brane tension in [59] expressing later time transition for smaller value of brane tension. In comparison to [58, 59], our model predicts later time for quark-hadron phase transition. In [60, 61], the authors studied the phase transition in RS brane-world including a bulk chameleon like scalar field. This model provided a non-conservation equation of state and their results expressed a phase transition at about nano-second after the Big Bang. However in our paper, phase transition occurs at about micro-second after the Big Bang and displays later transition time in comparison to [60, 61] for both QCD and first-order phase transition formalism. This differences is due to energy transition between bulk and brane, which is existed in [60, 61]. However results of the present paper is more consistence with other papers which there isn't any energy transition between bulk and brane.

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